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A CLASSICAL APPROACH TO THE INTUITIONISTIC FUZZY STABILITY OF MIXED UNDECIC- DODECIC FUNCTIONAL EQUATION

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ABSTRACT. In this paper, the authors introduce and establish the stability of single variable mixed undecic- dodecic functional equation in intuitionistic fuzzy Banach spaces.

1. INTRODUCTION

The study of stability problems for functional equations is connected to a question of Ulam [31] concerning the stability of group homomorphisms and positively answered for Banach spaces by Hyers [14]. It was further generalized and outstanding results was obtained by number of authors see ([15, 18, 23, 25, 26]). During the last seven decades, the above problem was tackled by numerous authors and its solutions via various forms of functional equations were discussed one can refer [11, 12, 13, 16, 20, 24, 32, 33, 35] and references cited there in.

M.Arunkumar et. al., [3] introduced and established the general solution and generalized Ulam - Hyers stability of the simple additive-quadratic and simple cubic-quartic functional equations

$$f(2x) = 3f(x) + f(-x), \tag{1.1}$$

and

$$g(2x) = 12g(x) + 4g(-x), \tag{1.2}$$

having solutions

$$f(x) = ax + bx^2 \qquad and \qquad g(x) = cx^3 + dx^4, \tag{1.3}$$

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By the above motivation, recently M. Arunkumar and P. Agilan introduced and discussed the generalized Ulam - Hyers stability simple mixed quintic - sextic, septic - octic, nonic - decic functional equations of the form

$$\vartheta(2p) = 48\vartheta(p) + 16\vartheta(-p) \tag{1.4}$$

$$\eta(2q) = 192\eta(q) + 64\eta(-q) \tag{1.5}$$

$$\psi(2r) = 768\psi(r) + 256\psi(-r) \tag{1.6}$$

in Banach space.

In this paper, the authors introduce and establish the stability of single variable mixed undecic - dodecic functional equation

$$g(5x) = 146,484,375g(x) + 97,656,250g(-x) \tag{1.7}$$

in intuitionistic fuzzy Banach spaces. The above equation having solution

$$g(x) = c_1\, x^{11} + c_2\, x^{12} \tag{1.8}$$

Lemma 1.1. *An odd function $g : X \to Y$ satisfies the functional equation (1.7) then*

$$g(5x) = 48,828,125g(x) = 5^{11}g(x)$$

for all $x \in X$

Lemma 1.2. *An even function $g : X \to Y$ satisfies the functional equation (1.7) then*

$$g(5x) = 244,140,625g(x) = 5^{12}g(x)$$

for all $x \in X$

2. Definitions and Notations of Intuitionistic Fuzzy Banach Space

Now, we recall the basic definitions and notations in the setting of intuitionistic fuzzy normed space.

Definition 2.1. *A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is said to be continuous t-norm if $*$ satisfies the following conditions:*

(1) *$*$ is commutative and associative;*
(2) *$*$ is continuous;*
(3) *$a * 1 = a$ for all $a \in [0,1]$;*
(4) *$a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.*

Definition 2.2. *A binary operation $\diamond : [0,1] \times [0,1] \longrightarrow [0,1]$ is said to be continuous t-conorm if $\diamond$ satisfies the following conditions:*

(1') *$\diamond$ is commutative and associative;*
(2') *$\diamond$ is continuous;*
(3') *$a \diamond 0 = a$ for all $a \in [0,1]$;*
(4') *$a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.*

Using the notions of continuous t-norm and t-conorm, Saadati and Park [28] introduced the concept of intuitionistic fuzzy normed space as follows:

Definition 2.3. *The five-tuple $(X, \mu, \nu, *, \diamond)$ is said to be an intuitionistic fuzzy normed space (for short, IFNS) if X is a vector space, $*$ is a continuous t-norm, $\diamond$ is a continuous $t-$ conorm, and μ, ν are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions. For every $x, y \in X$ and $s, t > 0$*

.

(IFN1) $\mu(x,t)+\nu(x,t)\leq 1$,
(IFN2) $\mu(x,t)>0$,
(IFN3) $\mu(x,t)=1$, *if and only if* $x=0$.
(IFN4) $\mu(\alpha x,t)=\mu\left(x,\frac{t}{\alpha}\right)$ *for each* $\alpha\neq 0$,
(IFN5) $\mu(x,t)*\mu(y,s)\leq\mu(x+y,t+s)$,
(IFN6) $\mu(x,\cdot):(0,\infty)\to[0,1]$ *is continuous*,
(IFN7) $\lim_{t\to\infty}\mu(x,t)=1$ *and* $\lim_{t\to 0}\mu(x,t)=0$,
(IFN8) $\nu(x,t)<1$,
(IFN9) $\nu(x,t)=0$, *if and only if* $x=0$.
(IFN10) $\nu(\alpha x,t)=\nu\left(x,\frac{t}{\alpha}\right)$ *for each* $\alpha\neq 0$,
(IFN11) $\nu(x,t)\diamond\nu(y,s)\geq\nu(x+y,t+s)$,
(IFN12) $\nu(x,\cdot):(0,\infty)\to[0,1]$ *is continuous*,
(IFN13) $\lim_{t\to\infty}\nu(x,t)=0$ *and* $\lim_{t\to 0}\nu(x,t)=1$.

In this case, (μ,ν) *is called an intuitionistic fuzzy norm.*

Example 2.4. *Let* $(X,\|\cdot\|)$ *be a normed space. Let* $a*b=ab$ *and* $a\diamond b=\min\{a+b,1\}$ *for all* $a,b\in[0,1]$. *For all* $x\in X$ *and every* $t>0$, *consider*

$$\mu(x,t)=\begin{cases}\frac{t}{t+\|x\|} & if \quad t>0;\\ 0 & if \quad t\leq 0;\end{cases}$$

and

$$\nu(x,t)=\begin{cases}\frac{\|x\|}{t+\|x\|} & if \quad t>0;\\ 0 & if \quad t\leq 0.\end{cases}$$

Then $(X,\mu,\nu,*,\diamond)$ *is an IFN-space.*

The concepts of convergence and Cauchy sequences in an intuitionistic fuzzy normed space are investigated in [28].

Definition 2.5. *Let* $(X,\mu,\nu,*,\diamond)$ *be an IFNS. Then, a sequence* $x=\{x_k\}$ *is said to be intuitionistic fuzzy convergent to a point* $L\in X$ *if*

$$\lim\ \mu(x_k-L,t)=1 \quad and \quad \lim\ \nu(x_k-L,t)=0$$

for all $t>0$. *In this case, we write*

$$x_k\xrightarrow{IF}L \qquad as \qquad k\to\infty$$

Definition 2.6. *Let* $(X,\mu,\nu,*,\diamond)$ *be an IFN-space. Then,* $x=\{x_k\}$ *is said to be intuitionistic fuzzy Cauchy sequence if*

$$\mu\left(x_{k+p}-x_k,t\right)=1 \qquad and \qquad \nu\left(x_{k+p}-x_k,t\right)=0$$

for all $t>0$, *and* $p=1,2\cdots$.

Definition 2.7. *Let* $(X,\mu,\nu,*,\diamond)$ *be an IFN-space. Then* $(X,\mu,\nu,*,\diamond)$ *is said to be complete if every intuitionistic fuzzy Cauchy sequence in* $(X,\mu,\nu,*,\diamond)$ *is intuitionistic fuzzy convergent* $(X,\mu,\nu,*,\diamond)$.

Hereafter throughout this paper, assume that X is a linear space, (Z,μ',ν') is an intuitionistic fuzzy normed space and (Y,μ,ν) an intuitionistic fuzzy Banach space.

3. Stability Results In Intuitionistic Fuzzy Banach Space

Theorem 3.1. *Let $j \in \{1,-1\}$. Let $\Psi_{UD} : X \longrightarrow Z$ be a function such that for some $0 < \left(\frac{k}{5^{11}}\right)^j < 1$,*

$$\left.\begin{aligned} \mu'\left(\Psi_{UD}\left(5^{nj}x\right),t\right) \geq \mu'\left(k^{nj}\Psi_{UD}(x),t\right) \\ \nu'\left(\Psi_{UD}\left(5^{nj}x\right),t\right) \leq \nu'\left(k^{nj}\Psi_{UD}(x),t\right) \end{aligned}\right\} \tag{3.1}$$

for all $x \in X$ and all $t > 0$ and

$$\left.\begin{aligned} \lim_{n\to\infty} \mu'\left(\Psi_{UD}\left(5^{jn}x\right),5^{11jn}t\right) = 1 \\ \lim_{n\to\infty} \nu'\left(\Psi_{UD}\left(5^{jn}x\right),5^{11jn}t\right) = 0 \end{aligned}\right\} \tag{3.2}$$

for all $x \in X$ and all $t > 0$. Let $g_u : X \to Y$ be an odd function satisfying the inequality

$$\left.\begin{aligned} \mu\left(g_u(5x) - 146,484,375g_u(x) - 97,656,250g_u(-x),t\right) \geq \mu'\left(\Psi_{UD}(x),t\right) \\ \nu\left(g_u(5x) - 146,484,375g_u(x) - 97,656,250g_u(-x),t\right) \leq \nu'\left(\Psi_{UD}(x),t\right) \end{aligned}\right\} \tag{3.3}$$

for all $x \in X$ and all $t > 0$. Then there exists a unique undecic mapping $\mathcal{U} : X \longrightarrow Y$ satisfying (1.7) and

$$\left.\begin{aligned} \mu\left(g_u(x) - \mathcal{U}(x),t\right) \geq \mu'\left(\Psi_{UD}(x),t|5^{11}-k|\right) \\ \nu\left(g_u(x) - \mathcal{U}(x),t\right) \leq \nu'\left(\Psi_{UD}(x),t|5^{11}-k|\right) \end{aligned}\right\} \tag{3.4}$$

for all $x \in X$ and all $t > 0$.

Proof. **Case (i):** Let $j = 1$. Using oddness of f in in (3.3), we obtain

$$\left.\begin{aligned} \mu\left(g_u(5x) - 48828125g(x),t\right) \geq \mu'\left(\Psi_{UD}(x),t\right) \\ \nu\left(g_u(5x) - 48828125g(x),t\right) \leq \nu'\left(\Psi_{UD}(x),t\right) \end{aligned}\right\} \tag{3.5}$$

for all $x \in X$ and all $t > 0$. From (3.5) we arrive

$$\left.\begin{aligned} \mu\left(g_u(5x) - 5^{11}g(x),t\right) \geq \mu'\left(\Psi_{UD}(x),t\right) \\ \nu\left(g_u(5x) - 5^{11}g(x),t\right) \leq \nu'\left(\Psi_{UD}(x),t\right) \end{aligned}\right\} \tag{3.6}$$

for all $x \in X$ and all $t > 0$. Using (IFN4) and (IFN10) in (3.6), we have

$$\left.\begin{aligned} \mu\Big(\frac{g_u(5x)}{5^{11}} - g_u(x), \frac{t}{5^{11}}\Big) \geq \mu'\left(\Psi_{UD}(x),t\right) \\ \nu\Big(\frac{g_u(5x)}{5^{11}} - g_u(x), \frac{t}{5^{11}}\Big) \leq \nu'\left(\Psi_{UD}(x),t\right) \end{aligned}\right\} \tag{3.7}$$

for all $x \in X$ and all $t > 0$. Substituting x by $5^n x$ in (3.7), we arrive

$$\left.\begin{aligned} \mu\Big(\frac{g_u(5^{(n+1)}x)}{5^{11}} - g_u(5^n x), \frac{t}{5^{11}}\Big) \geq \mu'\left(\Psi_{UD}(5^n x),t\right) \\ \nu\Big(\frac{g_u(5^{(n+1)}x)}{5^{11}} - g_u(5^n x), \frac{t}{5^{11}}\Big) \leq \nu'\left(\Psi_{UD}(5^n x),t\right) \end{aligned}\right\} \tag{3.8}$$

for all $x \in X$ and all $t > 0$. It is easy to verify from (3.8) and using (3.1), (IFN4), (IFN10) that

$$\left.\begin{aligned} \mu\Big(\frac{g_u(5^{(n+1)}x)}{5^{11(n+1)}} - \frac{g_u(5^n x)}{5^{11n}}, \frac{t}{5^{11}\,.\,5^{11n}}\Big) \geq \mu'\left(\Psi_{UD}(x), \frac{t}{p^n}\right) \\ \nu\Big(\frac{g_u(5^{n+1}x)}{5^{11(n+1)}} - \frac{g_u(5^n x)}{5^{11n}}, \frac{t}{5^{11}\,.\,5^{11n}}\Big) \leq \nu'\left(\Psi_{UD}(x), \frac{t}{p^n}\right) \end{aligned}\right\} \tag{3.9}$$

for all $x \in X$ and all $t > 0$. Interchanging t into $k^n t$ in (3.9), we have

$$\left.\begin{aligned} \mu\Big(\frac{g_u(5^{n+1}x)}{5^{11(n+1)}} - \frac{g_u(5^n x)}{5^{11n}}, \frac{t \cdot k^n}{5^{11}\,.\,5^{11n}}\Big) \geq \mu'\left(\Psi_{UD}(x), t\right) \\ \nu\Big(\frac{g_u(5^{n+1}x)}{5^{11(n+1)}} - \frac{g_u(5^n x)}{5^{11n}}, \frac{t \cdot k^n}{5^{11}\,.\,5^{11n}}\Big) \leq \nu'\left(\Psi_{UD}(x), t\right) \end{aligned}\right\} \tag{3.10}$$

for all $x \in X$ and all $t > 0$. It is easy to see that

$$\frac{g_u(5^n x)}{5^{11n}} - g_u(x) = \sum_{i=0}^{n-1} \frac{g_u(5^{i+1}x)}{5^{11(i+1)}} - \frac{g_u(5^i x)}{5^{11i}} \tag{3.11}$$

for all $x \in X$. It follows from (3.10) and (3.11), we get

$$\left.\begin{aligned} \mu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) = \mu\left(\sum_{i=0}^{n-1} \frac{g_u(5^{i+1}x)}{5^{11(i+1)}} - \frac{g_u(5^i x)}{5^{11i}}, \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) \\ \nu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) = \nu\left(\sum_{i=0}^{n-1} \frac{g_u(5^{i+1}x)}{5^{11(i+1)}} - \frac{g_u(5^i x)}{5^{i}}, \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) \end{aligned}\right\} \tag{3.12}$$

for all $x \in X$ and all $t > 0$. Using (IFNS5) and (IFNA11) in (3.12), we have

$$\left.\begin{aligned} \mu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), \sum_{i=0}^{n-1} \frac{k^i t}{5^{11}\,.\,5^{11i}}\right) \geq \prod_{i=0}^{n-1} \mu\left(\frac{g_u(5^{i+1}x)}{5^{11(i+1)}} - \frac{g_u(5^i x)}{5^{11i}}, \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) \\ \nu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), \sum_{i=0}^{n-1} \frac{k^i t}{5^{11}\,.\,5^{11i}}\right) \leq \prod_{i=0}^{n-1} \nu\left(\frac{g_u(5^{i+1}x)}{5^{11(i+1)}} - \frac{g_u(5^i x)}{5^{11i}}, \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) \end{aligned}\right\} \tag{3.13}$$

where

$$\prod_{i=0}^{n-1} c_j = c_1 * c_2 * \cdots * c_n$$

and

$$\prod_{i=0}^{n-1} d_j = d_1 \diamond d_2 \diamond \cdots \diamond d_n$$

for all $x \in X$ and all $t > 0$. Hence, from (3.13) and (3.10), we arrive

$$\left.\begin{aligned} \mu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) \geq \prod_{i=0}^{n-1} \mu'\left(\Psi_{UD}(x), t\right) = \mu'\left(\Psi_{UD}(x), t\right) \\ \nu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\,.\,5^{11i}}\right) \leq \prod_{i=0}^{n-1} \nu'\left(\Psi_{UD}(x), t\right) = \nu'\left(\Psi_{UD}(x), t\right) \end{aligned}\right\} \tag{3.14}$$

for all $x \in X$ and all $t > 0$. Replacing x by $5^m x$ in (3.14) and using (3.1), (IFN4), (IFN10), we obtain

$$\left.\begin{aligned} \mu\left(\frac{g_u(5^{n+m}x)}{5^{11(n+m)}} - \frac{g_u(5^m x)}{5^{11m}}, \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\, .\, 5^{11(i+m)}}\right) &\geq \mu'\left(\Psi_{UD}(5^m x), t\right) = \mu'\left(\Psi_{UD}(x), \tfrac{t}{k^m}\right) \\ \nu\left(\frac{g_u(5^{n+m}x)}{5^{11(n+m)}} - \frac{g_u(5^m x)}{5^{11m}}, \sum_{i=0}^{n-1} \frac{k^i\, t}{5^{11}\, .\, 5^{11(i+m)}}\right) &\leq \nu'\left(\Psi_{UD}(5^m x), t\right) = \nu'\left(\Psi_{UD}(x), \tfrac{t}{k^m}\right) \end{aligned}\right\} \quad (3.15)$$

for all $x \in X$ and all $t > 0$ and all $m, n \geq 0$. Replacing t by $k^m t$ in (3.15), we get

$$\left.\begin{aligned} \mu\left(\frac{g_u(5^{n+m}x)}{5^{11(n+m)}} - \frac{g_u(5^m x)}{5^{11m}}, \sum_{i=0}^{n-1} \frac{k^{i+m}\, t}{5^{11}\, .\, 5^{11(i+m)}}\right) &\geq \mu'\left(\Psi_{UD}(x), t\right) \\ \nu\left(\frac{g_u(5^{n+m}x)}{5^{11(n+m)}} - \frac{g_u(5^m x)}{5^{11m}}, \sum_{i=0}^{n-1} \frac{k^{i+m}\, t}{5^{11}\, .\, 5^{11(i+m)}}\right) &\leq \nu'\left(\Psi_{UD}(x), t\right) \end{aligned}\right\} \quad (3.16)$$

for all $x \in X$ and all $t > 0$ and all $m, n \geq 0$. The relation (3.16) implies that

$$\left.\begin{aligned} \mu\left(\frac{g_u(5^{n+m}x)}{5^{11(n+m)}} - \frac{g_u(5^m x)}{5^{11m}}, t\right) &\geq \mu'\left(\Psi_{UD}(x), \frac{t}{\sum_{i=m}^{n-1} \frac{p^i}{5^{11}.5^{11i}}}\right) \\ \nu\left(\frac{g_u(5^{n+m}x)}{5^{11(n+m)}} - \frac{g_u(5^m x)}{5^{11m}}, t\right) &\leq \nu'\left(\Psi_{UD}(x), \frac{t}{\sum_{i=m}^{n-1} \frac{p^i}{5^{11}.5^{11i}}}\right) \end{aligned}\right\} \quad (3.17)$$

holds for all $x \in X$ and all $t > 0$ and all $m, n \geq 0$. Since $0 < k < 5^{11}$ and $\sum_{i=0}^{n} \left(\frac{k}{5^{11}}\right)^i < \infty$. The Cauchy criterion for convergence in IFNS shows that the sequence $\left\{\frac{g_u(5^n x)}{5^{11n}}\right\}$ is Cauchy in (Y, μ, ν). Since (Y, μ, ν) is a complete IFN-space this sequence converges to some point $\mathcal{U}(x) \in Y$. So, one can define the mapping $\mathcal{U} : X \longrightarrow Y$ by

$$\lim_{n\to\infty} \mu\left(\frac{g_u(5^n x)}{5^{11n}} - \mathcal{U}(x), t\right) = 1,$$

$$\lim_{n\to\infty} \nu\left(\frac{g_u(5^n x)}{5^{11n}} - \mathcal{U}(x), t\right) = 0$$

for all $x \in X$ and all $t > 0$. Hence

$$\frac{g_u(5^n x)}{5^{11n}} \xrightarrow{IF} \mathcal{U}(x), \quad as \quad n \to \infty.$$

Letting $m = 0$ in (3.16), we arrive

$$\left.\begin{aligned} \mu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), t\right) &\geq \mu'\left(\Psi(x), \frac{t}{\sum_{i=0}^{n-1} \frac{k^i}{5^{11}.5^{11i}}}\right) \\ \nu\left(\frac{g_u(5^n x)}{5^{11n}} - g_u(x), t\right) &\leq \nu'\left(\Psi(x), \frac{t}{\sum_{i=0}^{n-1} \frac{k^i}{5^{11}.5^{11i}}}\right) \end{aligned}\right\} \quad (3.18)$$

for all $x \in X$ and all $t > 0$. Letting $n \to \infty$ in (3.18), we arrive

$$\left.\begin{aligned} \mu\left(\mathcal{U}(x) - g_u(x), t\right) \geq \mu'\left(\Psi_{UD}(x), t|5^{11} - k|\right) \\ \nu\left(\mathcal{U}(x) - g_u(x), t\right) \leq \nu'\left(\Psi_{UD}(x), t|5^{11} - k|\right) \end{aligned}\right\} \tag{3.19}$$

for all $x \in X$ and all $t > 0$. To prove f satisfies (1.7), replacing x by $5^n x$ in (3.3) respectively, we obtain

$$\left.\begin{aligned} &\mu\left(\tfrac{1}{5^{11n}}\left[g_u(5\cdot 5^n x) - 146,484,375 g_u(5^n x) - 97,656,250 g_u(-5^n x)\right], t\right) \\ &\qquad\qquad \geq \mu'\left(\Delta_{UD}(5^n x), 5^{11n} t\right) \\ &\nu\left(\tfrac{1}{5^{11n}}\left[g_u(5\cdot 5^n x) - 146,484,375 g_u(5^n x) - 97,656,250 g_u(-5^n x)\right], t\right) \\ &\qquad\qquad \geq \nu'\left(\Delta_{UD}(5^n x), 5^{11n} t\right) \end{aligned}\right\} \tag{3.20}$$

for all $x \in X$ and all $t > 0$. Now,

$$\begin{aligned} &\mu\Big(\mathcal{U}(5x) - 146,484,375\mathcal{U}(x) - 97,656,250\mathcal{U}(-x), t\Big) \\ &\geq \mu\Big(\mathcal{U}(5x) - \frac{1}{5^{11n}} g_u(5x), \frac{t}{4}\Big) \\ &\quad * \mu\Big(-146,484,375\mathcal{U}(x) + 146,484,375\frac{1}{5^{11n}} g_u(x), \frac{t}{4}\Big) \\ &\quad * \mu\Big(-97,656,250\mathcal{U}(-x) + 97,656,250\frac{1}{5^{11n}} g_u(-x), \frac{t}{4}\Big) \\ &\quad * \mu\Big(\frac{1}{5^{11n}} g_u(5x) - 146,484,375\frac{1}{5^{11n}} g_u(x) - 97,656,250\frac{1}{5^{11n}} g_u(-x), \frac{t}{4}\Big) \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} &\nu\Big(\mathcal{U}(5x) - 146,484,375\mathcal{U}(x) - 97,656,250\mathcal{U}(-x), t\Big) \\ &\geq \nu\Big(-146,484,375\mathcal{U}(x) + 146,484,375\frac{1}{5^{11n}} g_u(x), \frac{t}{4}\Big) \\ &\quad \diamond \mu\Big(-146,484,375\mathcal{U}(x) + 146,484,375\frac{1}{5^{11n}} g_u(x), \frac{t}{4}\Big) \\ &\quad \diamond \nu\Big(-97,656,250\mathcal{U}(-x) + 97,656,250\frac{1}{5^{11n}} g_u(-x), \frac{t}{4}\Big) \\ &\quad \diamond \nu\Big(\frac{1}{5^{11n}} g_u(5x) - 146,484,375\frac{1}{5^{11n}} g_u(x) - 97,656,250\frac{1}{5^{11n}} g_u(-x), \frac{t}{4}\Big) \end{aligned} \tag{3.22}$$

for all $x \in X$ and all $t > 0$. Also,

$$\left.\begin{aligned} \lim_{n\to\infty} \mu\Big(\tfrac{1}{5^{11n}}\left[g_u(5\cdot 5^n x) - 146,484,375 g_u(5^n x) - 97,656,250 g_u(-5^n x)\right], \tfrac{t}{4}\Big) = 1 \\ \lim_{n\to\infty} \nu\Big(\tfrac{1}{5^{11n}}\left[g_u(5\cdot 5^n x) - 146,484,375 g_u(5^n x) - 97,656,250 g_u(-5^n x)\right], \tfrac{t}{4}\Big) = 0 \end{aligned}\right\} \tag{3.23}$$

for all $x \in X$ and all $t > 0$. Letting $n \to \infty$ in (3.21), (3.22) and using (3.23), we find that $\mathcal{U}$ fulfills (1.7). Therefore, $\mathcal{U}$ is a undecic mapping. In order to prove $\mathcal{U}(x)$ is unique, let $\mathcal{U}'(x)$ be another undecic functional equation satisfying (1.7) and (3.4). Hence,

$$\begin{aligned}
&\mu(\mathcal{U}(x)-\mathcal{U}'(x),t)\\
&\geq \mu\left(\mathcal{U}(5^n x)-g_u(5^n x),\frac{t.5^{11n}}{2}\right)*\mu\left(g_u(5^n x)-\mathcal{U}'(5^n x),\frac{t.5^{11n}}{2}\right)\\
&\geq \mu'\left(\Psi_{UD}(5^n x),\frac{5^{11n}\ t\ |5^{11}-k|}{2}\right)\geq \mu'\left(\Psi_{UD}(x),\frac{5^{11n}\ t\ |5^{11}-k|}{2\cdot k^n}\right)\\
&\nu(\mathcal{U}(x)-\mathcal{U}'(x),t)\\
&\leq \nu\left(\mathcal{U}(5^n x)-g_u(5^n x),\frac{t.5^{11n}}{2}\right)\diamond\nu\left(g_u(5^n x)-\mathcal{U}'(5^n x),\frac{t.5^{11n}}{2}\right)\\
&\leq \nu'\left(\Psi_{UD}(5^n x),\frac{5^{11n}t\ |5^{11}-k|}{2}\right)\leq \nu'\left(\Psi_{UD}(x),\frac{5^{11n}\ t\ |5^{11}-k|}{2\cdot k^n}\right)
\end{aligned}$$

for all $x\in X$ and all $t>0$. Since $\lim_{n\to\infty}\frac{5^{11n}\ t\ |5^{11}-k|}{2\ k^n}=\infty$, we obtain

$$\left.\begin{array}{l}\lim_{n\to\infty}\mu'\left(\Psi_{UD}(x),\frac{5^{11n}\ t\ |5^{11}-k|}{2\cdot k^n}\right)=1\\ \lim_{n\to\infty}\nu'\left(\Psi_{UD}(x),\frac{5^{11n}\ t\ |5^{11}-k|}{2\cdot k^n}\right)=0\end{array}\right\}$$

for all $x\in X$ and all $t>0$. Thus

$$\left.\begin{array}{l}\mu(\mathcal{U}(x)-\mathcal{U}'(x),t)=1\\ \nu(\mathcal{U}(x)-\mathcal{U}'(x),t)=0\end{array}\right\}$$

for all $x\in X$ and all $t>0$. Hence, $\mathcal{U}(x)=\mathcal{U}'(x)$. Therefore, $\mathcal{U}(x)$ is unique.
Case 2: For $j=-1$. Putting x by $\frac{x}{5}$ in (3.6), we get

$$\left.\begin{array}{l}\mu\left(g_u(x)-5^{11}g\left(\frac{x}{5}\right),t\right)\geq\mu'\left(\Psi_{UD}\left(\frac{x}{5}\right),t\right)\\ \nu\left(g_u(x)-5^{11}g\left(\frac{x}{5}\right),t\right)\leq\nu'\left(\Psi_{UD}\left(\frac{x}{5}\right),t\right)\end{array}\right\}\tag{3.24}$$

for all $x\in X$ and all $t>0$. The rest of the proof is similar to that of Case 1. This completes the proof. □

The following corollary is an immediate consequence of Theorem 3.1, regarding the stability of (1.7)

Corollary 3.2. *Suppose that an odd function $g_u:X\longrightarrow Y$ satisfies the double inequality*

$$\left.\begin{array}{l}\mu\left(g_u(5x)-146,484,375g_u(x)-97,656,250g_u(-x),t\right)\geq\left\{\begin{array}{l}\mu'\left(\Pi,t\right),\\ \mu'\left(\Pi\left(||x||^r\right),t\right),\end{array}\right.\\ \nu\left(g_u(5x)-146,484,375g_u(x)-97,656,250g_u(-x),t\right)\leq\left\{\begin{array}{l}\nu'\left(\Pi,t\right),\\ \nu'\left(\Pi\left(||x||^r\right),t\right),\end{array}\right.\end{array}\right\}\tag{3.25}$$

for all $x\in X$ and all $t>0$, where Π,r are constants with $\Pi>0$ and $r\neq 11$. Then there exists a unique udecic mapping $\mathcal{U}:X\longrightarrow Y$ such that

$$\left.\begin{array}{l}\mu\left(g_u(x)-\mathcal{U}(x),t\right)\geq\left\{\begin{array}{l}\mu'\left(\Pi,|5^{11}-1|t\right),\\ \mu'\left(\Pi||x||^r,|5^{11}-5^r|t\right),\end{array}\right.\\ \nu\left(g_u(x)-\mathcal{U}(x),t\right)\leq\left\{\begin{array}{l}\nu'\left(\Pi,|5^{11}-1|t\right),\\ \mu'\left(\Pi||x||^r,|5^{11}-5^r|t\right),\end{array}\right.\end{array}\right\}\tag{3.26}$$

for all $x\in X$ and all $t>0$.

Theorem 3.3. *Let $j \in \{1, -1\}$. Let $\Delta_{UD} : X \longrightarrow Z$ be a function such that for some $0 < \left(\frac{k}{5^{12}}\right)^j < 1$,*

$$\left.\begin{array}{l} \mu'\left(\Psi_{UD}\left(5^{nj}x\right), t\right) \geq \mu'\left(k^{nj}\Psi_{UD}(x), t\right) \\ \nu'\left(\Psi_{UD}\left(5^{nj}x\right), t\right) \leq \nu'\left(k^{nj}\Delta_{UD}(x), t\right) \end{array}\right\} \tag{3.27}$$

for all $x \in X$ and all $t > 0$ and

$$\left.\begin{array}{l} \lim_{n\to\infty} \mu'\left(\Psi_{UD}\left(5^{jn}x\right), 5^{12jn}t\right) = 1 \\ \lim_{n\to\infty} \nu'\left(\Psi_{UD}\left(5^{jn}x\right), 5^{12jn}t\right) = 0 \end{array}\right\} \tag{3.28}$$

for all $x \in X$ and all $t > 0$. Let $g_d : X \to Y$ be an even function satisfying the inequality

$$\left.\begin{array}{l} \mu\left(g_d(5x) - 146,484,375g_d(x) - 97,656,250g_d(-x), t\right) \geq \mu'\left(\Psi_{UD}(x), t\right) \\ \nu\left(g_d(5x) - 146,484,375g_d(x) - 97,656,250g_d(-x), t\right) \leq \nu'\left(\Psi_{UD}(x), t\right) \end{array}\right\} \tag{3.29}$$

for all $x \in X$ and all $t > 0$. Then there exists a unique dodecic mapping $\mathcal{D} : X \longrightarrow Y$ satisfying (1.7) and

$$\left.\begin{array}{l} \mu\left(g_d(x) - \mathcal{D}(x), t\right) \geq \mu'\left(\Psi_{UD}(x), |5^{12} - k|t\right) \\ \nu\left(g_d(x) - \mathcal{U}(x), t\right) \leq \nu'\left(\Psi_{UD}(x), |5^{12} - k|t\right) \end{array}\right\} \tag{3.30}$$

for all $x \in X$ and all $t > 0$.

Proof. **Case (i):** Let $j = 1$. Using evenness of f in in (3.29), we obtain

$$\left.\begin{array}{l} \mu\left(g_d(5x) - 244140625g(x), t\right) \geq \mu'\left(\Psi_{UD}(x), t\right) \\ \nu\left(g_d(5x) - 244140625g(x), t\right) \leq \nu'\left(\Psi_{UD}(x), t\right) \end{array}\right\} \tag{3.31}$$

for all $x \in X$ and all $t > 0$. From (3.31) we have

$$\left.\begin{array}{l} \mu\left(g_d(5x) - 5^{12}g(x), t\right) \geq \mu'\left(\Psi_{UD}(x), t\right) \\ \nu\left(g_d(5x) - 5^{12}g(x), t\right) \leq \nu'\left(\Psi_{UD}(x), t\right) \end{array}\right\} \tag{3.32}$$

for all $x \in X$ and all $t > 0$. The rest of the proof is similar to that of Theorem 3.1. □

The following corollary is an immediate consequence of Theorem 3.3, regarding the stability of (1.7)

Corollary 3.4. *Suppose that an even function $g_d : X \longrightarrow Y$ satisfies the double inequality*

$$\left.\begin{array}{l} \mu\left(g_d(5x) - 146,484,375g_d(x) - 97,656,250g_d(-x), t\right) \geq \left\{ \begin{array}{l} \mu'(\Pi, t), \\ \mu'(\Pi(||x||^r), t), \end{array}\right. \\ \nu\left(g_d(5x) - 146,484,375g_d(x) - 97,656,250g_d(-x), t\right) \leq \left\{ \begin{array}{l} \nu'(\Pi, t), \\ \nu'(\Pi(||x||^r), t), \end{array}\right. \end{array}\right\} \tag{3.33}$$

for all $x \in X$ and all $t > 0$, where Π, r are constants with $\Pi > 0$ and $r \neq 12$. Then there exists a unique dodecic mapping $\mathcal{D} : X \longrightarrow Y$ such that

$$\left.\begin{aligned} \mu\left(g_d(x) - \mathcal{D}(x), t\right) &\geq \begin{cases} \mu'\left(\Pi, |5^{12} - 1|t\right), \\ \mu'\left(\Pi||x||^r, |5^{12} - 5^r|t\right), \end{cases} \\ \nu\left(g_d(x) - \mathcal{D}(x), t\right) &\leq \begin{cases} \nu'\left(\Pi, |5^{12} - 1|t\right), \\ \mu'\left(\Pi||x||^r, |5^{12} - 5^r|t\right), \end{cases} \end{aligned}\right\} \tag{3.34}$$

for all $x \in X$ and all $t > 0$.

Theorem 3.5. *Let $j \in \{1, -1\}$. Let $\Delta_{UD} : X \longrightarrow Z$ be a function such that for some $0 < \left(\frac{k}{5^{11}}\right)^j, 0 < \left(\frac{k}{5^{12}}\right)^j < 1$, with conditions (3.1), (3.27), (3.2) and (3.28) for all $x \in X$ and all $t > 0$. Let $g : X \to Y$ be a function satisfying the inequality*

$$\left.\begin{aligned} \mu\left(g_d(5x) - 146,484,375 g_d(x) - 97,656,250 g_d(-x), t\right) &\geq \mu'\left(\Psi_{UD}(x), t\right) \\ \nu\left(g_d(5x) - 146,484,375 g_d(x) - 97,656,250 g_d(-x), t\right) &\leq \nu'\left(\Psi_{UD}(x), t\right) \end{aligned}\right\} \tag{3.35}$$

for all $x \in X$ and all $t > 0$. Then there exists a unique undecic mapping $\mathcal{U} : X \longrightarrow Y$ and a unique dodecic mapping $\mathcal{D} : X \longrightarrow Y$ satisfying (1.7) and

$$\left.\begin{aligned} &\mu\left(g(x) - \mathcal{U}(x) - \mathcal{D}(x), t\right) \\ &\geq \mu'\left(\Psi_{UD}(x), |5^{11} - p|t\right) * \mu'\left(\Psi_{UD}(-x), |5^{11} - p|t\right) \\ &\qquad * \mu'\left(\Psi_{UD}(x), |5^{12} - p|t\right) * \mu'\left(\Psi_{UD}(-x), |5^{12} - p|t\right) \\ &\nu\left(g(x) - \mathcal{U}(x) - \mathcal{D}(x), t\right) \\ &\leq \nu'\left(\Psi_{UD}(x), |5^{11} - p|t\right) \diamond \nu'\left(\Psi_{UD}(-x), |5^{11} - p|t\right) \\ &\qquad \diamond \nu'\left(\Psi_{UD}(x), |5^{12} - p|t\right) \diamond \nu'\left(\Psi_{UD}(-x), |5^{12} - p|t\right) \end{aligned}\right\} \tag{3.36}$$

for all $x \in X$ and all $t > 0$.

Proof. Let $g_o(x) = \dfrac{g_u(x) - g_u(-x)}{2}$ for all $x \in X$. Then $g_o(0) = 0$ and $g_o(-x) = -g_o(x)$ for all $x \in X$. Hence by Theorem 3.1, we have

$$\left.\begin{aligned} \mu\left(g_o(x) - \mathcal{U}(x), t\right) &\geq \mu'\left(\Psi_{UD}(x), |5^{11} - k|t\right) * \mu'\left(\Psi_{UD}(-x), |5^{11} - k|t\right) \\ \nu\left(g_o(x) - \mathcal{U}(x), t\right) &\leq \nu'\left(\Psi_{UD}(x), |5^{11} - k|t\right) \diamond \nu'\left(\Psi_{UD}(-x), |5^{11} - k|t\right) \end{aligned}\right\} \tag{3.37}$$

for all $x \in X$ and all $t > 0$. Also, let $g_e(x) = \dfrac{g_d(x) + g_d(-x)}{2}$ for all $x \in X$. Then $g_e(0) = 0$ and $g_e(-x) = g_e(x)$ for all $x \in X$. Hence by Theorem 3.3, we have

$$\left.\begin{aligned} \mu\left(g_e(x) - \mathcal{D}(x), t\right) &\geq \mu'\left(\Psi_{UD}(x), |5^{12} - k|t\right) * \mu'\left(\Psi_{UD}(-x), |5^{12} - k|t\right) \\ \nu\left(g_e(x) - \mathcal{D}(x), t\right) &\leq \nu'\left(\Psi_{UD}(x), |5^{12} - k|t\right) \diamond \nu'\left(\Psi_{UD}(-x), |5^{12} - k|t\right) \end{aligned}\right\} \tag{3.38}$$

for all $x \in X$ and all $t > 0$. Define

$$g(x) = g_o(x) + g_e(x) \tag{3.39}$$

for all $x \in X$. From (3.37),(3.38) and (3.39), we arrive

$$\begin{aligned}\mu\left(g(x)-\mathcal{U}(x)-\mathcal{D}(x),2t\right) &= \mu\left(g_o(x)+g_e(x)-\mathcal{U}(x)-\mathcal{D}(x),2t\right)\\ &\geq \mu\left(g_o(x)-\mathcal{U}(x),t\right)*\mu\left(g_e(x)-\mathcal{D}(x),t\right)\\ &\geq \mu'\left(\Psi_{UD}\left(x\right),|5^{11}-k|t\right)*\mu'\left(\Psi_{UD}\left(-x\right),|5^{11}-k|t\right)\\ &\quad *\mu'\left(\Psi_{UD}\left(x\right),|5^{12}-k|t\right)*\mu'\left(\Psi_{UD}\left(-x\right),|5^{12}-k|t\right)\end{aligned}$$

and

$$\begin{aligned}\nu\left(g(x)-\mathcal{U}(x)-\mathcal{D}(x),2t\right) &= \nu\left(g_o(x)+g_e(x)-\mathcal{U}(x)-\mathcal{D}(x),2t\right)\\ &\leq \nu\left(g_o(x)-\mathcal{U}(x),t\right)*\nu\left(g_e(x)-\mathcal{D}(x),t\right)\\ &\leq \nu'\left(\Psi_{UD}\left(x\right),|5^{11}-k|t\right)\diamond\nu'\left(\Delta_{UD}\left(-x\right),|5^{11}-k|t\right)\\ &\quad \diamond\nu'\left(\Psi_{UD}\left(x\right),|5^{12}-k|t\right)\diamond\nu'\left(\Psi_{UD}\left(-x\right),|5^{12}-k|t\right)\end{aligned}$$

for all $x \in X$ and all $t > 0$. □

The following corollary is an immediate consequence of Theorem 3.5, regarding the stability of (1.7)

Corollary 3.6. *Suppose that a function $g : X \longrightarrow Y$ satisfies the double inequality*

$$\left.\begin{aligned}\mu\left(g(5x)-146,484,375g(x)-97,656,250g(-x),t\right) &\geq \begin{cases}\mu'\left(\Pi,t\right),\\ \mu'\left(\Pi\left(||x||^r\right),t\right),\end{cases}\\ \nu\left(g(5x)-146,484,375g(x)-97,656,250g(-x),t\right) &\leq \begin{cases}\nu'\left(\Pi,t\right),\\ \nu'\left(\Pi\left(||x||^r\right),t\right),\end{cases}\end{aligned}\right\} \tag{3.40}$$

for all $x \in X$ and all $t > 0$, where Π, r are constants with $\Pi > 0$ and $r \neq 11, 12$. Then there exists a unique undecic mapping $\mathcal{U} : X \longrightarrow Y$ and a unique dodecic mapping $\mathcal{D} : X \longrightarrow Y$ such that

$$\left.\begin{aligned}&\mu\left(g(x)-\mathcal{U}(x)-\mathcal{D}(x),t\right)\\ &\geq \begin{cases}\mu'\left(\Pi,|5^{11}-1|t\right)*\mu'\left(\Pi,|5^{12}-1|t\right),\\ \mu'\left(\Pi||x||^r,|5^{11}-5^r|t\right)*\mu'\left(\Pi||x||^r,|5^{12}-5^r|t\right),\end{cases}\\ &\nu\left(g(x)-\mathcal{U}(x)-\mathcal{D}(x),t\right)\\ &\leq \begin{cases}\nu'\left(\Pi,|5^{11}-1|t\right)\diamond\nu'\left(\Pi,|5^{12}-1|t\right),\\ \nu'\left(\Pi||x||^r,|5^{11}-5^r|t\right)\diamond\nu'\left(\Pi||x||^r,|5^{12}-5^r|t\right),\end{cases}\end{aligned}\right\} \tag{3.41}$$

for all $x \in X$ and all $t > 0$.

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